

*Preprint submitted to Elsevier Science*1 February 2008

arXiv:quant-ph/0605016v1 1 May 2006

Quantum manipulation and simulation using

Josephson junction arrays

Xingxiang Zhou and Ari Mizel

Department of Physics, The Pennsylvania State University, University Park, PA

16802, USA

Abstract

We discuss the prospect of using quantum properties of large scale Josephson junc-

tion arrays for quantum manipulation and simulation. We study the collective vi-

brational quantum modes of a Josephson junction array and show that they provide

a natural and practical method for realizing a high quality cavity for superconduct-

ing qubit based QED. We further demonstrate that by using Josephson junction

arrays we can simulate a family of problems concerning spinless electron-phonon

and electron-electron interactions. These protocols require no or few controls over

the Josephson junction array and are thus relatively easy to realize given currently

available technology.

Key words: Qubit, Quantum computing, Josephson junction array

PACS: 03.67.Lx, 74.90.+n, 85.25.Dq, 85.25.Cp

Email

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dresses:

(Xingx-

ang

Zhou),

ari@phys.psu.edu

(Ari

Mizel).

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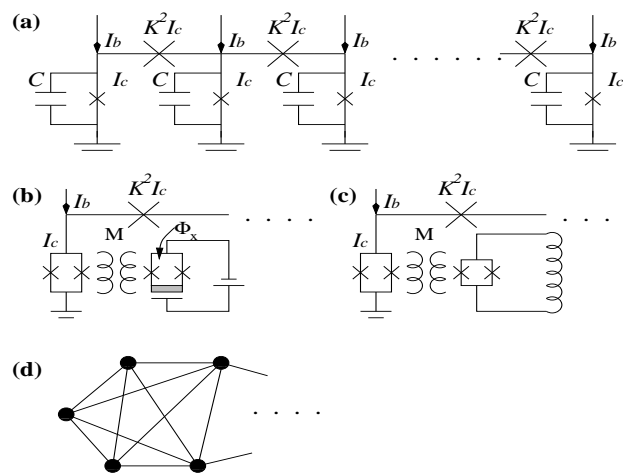


Fig. 1.

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A simple Josephson junction array consisting of N vertical and $N-1$ horizontal junctions.

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$$V = -E_J \sum_{i=0}^{N-1} (i_b \theta_i + \cos \theta_i) - K^2 E_J \sum_{i=0}^{N-2} \cos (\theta_i - \theta_{i+1}), (1)$$

where

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$$I_b/I_c.$$

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$$\frac{(V)_{ij}}{E_J K^2} = \delta_{ij} (2 + \cos \theta^{(0)} / K^2 - \delta_{i,0} - \delta_{i,N-1}) - \delta_{i,j-1} - \delta_{i,j+1}. (2)$$

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1146

di-

1147

vid-

1148

ual

1149

junc-

1150

tions.

1151

There-

1152

fore,

1153

this

1154

ap-

1155

proach

1156

al-

1157

lows

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us

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to

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take

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ad-

1162

van-

1163

tage

1164

of

1165

the

1166

high

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qual-

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ity

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Joseph-

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son

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junc-

1172

tions

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re-

1174

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for

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1178

per-

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con-

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duct-

1181

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1182

qubits

1183

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1188

high

1189

qual-

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ity

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onator.

1193

Note

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in

1195

the

1196

above

1197

we

1198

have

1199

as-

1200

sumed

1201

that

1202

all

1203

Joseph-

1204

son

1205

junc-

1206

tions

1207

in

1208

the

1209

ar-

1210

ray

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are

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iden-

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ti-

1214

cal.

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In

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re-

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ity,

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be

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case

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due

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to

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un-

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avoid-

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fab-

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ri-

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ca-

1233

tion

1234

er-

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fors.

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How-

1237

ever

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the

1239

crit-

1240

i-

1241

cal

1242

cur-

1243

rents

1244

of

1245

the

1246

junc-

1247

tions

1248

can

1249

be

1250

tuned

1251

by

1252

a

1253

mag-

1254

netic

1255

field

1256

so

1257

that

1258

the

1259

ef-

1260

fec-

1261

tive

1262

Joseph-

1263

son

1264

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1265

er-

1266

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of

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the

1269

junc-

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tions

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me-

1284

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1285

can

1286

be

1287

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1288

ti-

1289

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by

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per-

1292

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1293

ba-

1294

tion

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the-

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ory.

1297

As

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long

1299

as

1300

the

1301

am-

1302

pli-

1303

tude

1304

of

1305

the

1306

tran-

1307

si-

1308

tion

1309

ma-

1310

trix

1311

el-

1312

e-

1313

ment

1314

due

1315

to

1316

the

1317

asym-

1318

me-

1319

try

1320

is

1321

much

1322

smaller

1323

than

1324

the

1325

en-

1326

ergy

1327

gap

1328

be-

1329

tween

1330

the

1331

cen-

1332

ter

1333

of

1334

mass

1335

mo-

1336

tion

1337

mode

1338

and

1339

higher

1340

modes,

1341

the

1342

en-

1343

ergy

1344

and

1345

wave-

1346

func-

1347

tion

1348

of

1349

the

1350

cen-

1351

ter

1352

of

1353

mass

1354

mo-

1355

tion

1356

mode

1357

re-

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main

1359

close

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to

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un-

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per-

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turbed.

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In

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or-

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der

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to

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im-

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ple-

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ment

1371

pro-

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to-

1373

cols

1374

de-

1375

vel-

1376

oped

1377

in

1378

su-

1379

per-

1380

con-

1381

duct-

1382

ing

1383

qubit

1384

based

1385

cavity-

1386

QED

1387

[13,14,15,16],

1388

we

1389

need

1390

to

1391

cou-

1392

ple

1393

the

1394

su-

1395

per-

1396

con-

1397

duct-

1398

ing

1399

qubits

1400

to

1401

the

1402

cen-

1403

ter

1404

of

1405

mass

1406

mo-

1407

tion

1408

mode

1409

of

1410

the

1411

junc-

1412

tion

1413

ar-

1414

ray

1415

in

1416

a

1417

way

1418

such

1419

that

1420

they

1421

can

1422

ex-

1423

change

1424

en-

1425

ergy.

1426

This

1427

is

1428

shown

1429

in

Fig.

1431

1

1432

(b)

1433

and

1434

(c)

1435

for

1436

both

1437

charge

1438

and

1439

flux

1440

qubits.

1441

Here

1442

each

1443

ver-

1444

ti-

1445

cal

1446

junc-

1447

tion

1448

in

1449

the

1450

junc-

1451

tion

1452

ar-

1453

ray

1454

is

1455

re-

1456

placed

1457

by

1458

a

1459

small

1460

self-

1461

inductance

1462

dc-

1463

SQUID

1464

and

1465

cou-

1466

pled

1467

in-

1468

duc-

1469

tively

1470

to

1471

a

1472

su-

1473

per-

1474

con-

1475

duct-

1476

ing

1477

qubit.

1478

Con-

1479

sider

1480

the

1481

charge

1482

qubit

1483

whose

1484

Hamil-

1485

to-

1486

nian

1487

is

1488

$$H_Q =$$

$$-B^z\sigma^z/2-$$

1490

$$B^x \sigma^x / 2,$$

1491

where

1492

B^z

1493

and

1494

B^x

1495

are

1496

de-

1497

ter-

1498

mined

1499

by

1500

the

1501

gate

1502

volt-

1503

age

1504

and

1505

Joseph-

1506

son

1507

en-

1508

ergy

1509

of

1510

the

1511

charge

1512

qubit.

1513

When

1514

its

1515

en-

1516

ergy

1517

B^z

1518

is

1519

tuned

1520

close

1521

to

1522

$\nu_0 =$

1523

ω_p

1524

and

1525

its

1526

dc-

1527

SQUID

1528

is

1529

bi-

1530

ased

1531

at

1532

$\Phi_0/2$

1533

(in-

1534

clud-

1535

ing

1536

the

1537

flux

1538

due

1539

to

1540

the

1541

junc-

1542

tion

1543

ar-

1544

ray's

1545

bias

1546

cur-

1547

rent

1548

$I_b)$,

1549

the

1550

in-

1551

duc-

1552

tive

1553

cou-

1554

pling

1555

re-

1556

sults

1557

in

1558

a

1559

cou-

1560

pling

1561

Hamil-

1562

to-

1563

nian

1564

$$H_c =$$

1565

$$-g(a\sigma^++$$

$$a^\dagger \sigma^-),$$

1567

where

1568

$g =$

$$(M/2)(I_c \cos \theta^{(0)}) I_c^Q (2\pi/\Phi_0) \sqrt{\hbar/2C\omega_p N},$$

1570

M

1571

is

1572

the

1573

mu-

1574

tual

1575

in-

1576

duc-

1577

tance,

1578

I_c^Q

1579

is

1580

the

1581

crit-

1582

i-

1583

cal

1584

cur-

1585

rent

1586

of

1587

the

1588

dc-

1589

SQUID

1590

junc-

1591

tions

1592

of

1593

the

1594

charge

1595

qubit,

1596

and

1597

a

1598

is

1599

the

1600

an-

1601

ni-

1602

hi-

1603

la-

1604

tion

1605

op-

1606

er-

1607

a-

1608

tor

1609

for

1610

the

1611

cen-

1612

ter

1613

of

1614

mass

1615

mo-

1616

tion

1617

mode

1618

of

1619

the

1620

junc-

1621

tion

1622

ar-

1623

ray.

1624

In

1625

de-

1626

riv-

1627

ing

1628

$H_c,$

1629

we

1630

have

1631

used

1632

the

1633

ro-

1634

tat-

1635

ing

1636

wave

1637

ap-

1638

prox-

1639

i-

1640

ma-

1641

tion

1642

to

1643

drop

1644

terms

1645

that

1646

OS-

1647

cil-

1648

late

1649

at

1650

high

1651

fre-

1652

quen-

1653

cies.

1654

For

1655

the

1656

flux

1657

qubit

1658

case

1659

shown

1660

in

1661

Fig.

1662

1

1663

(b),

1664

we

1665

can

1666

de-

1667

rive

1668

the

1669

same

1670

cou-

1671

pling

1672

Hamil-

1673

to-

1674

nian,

1675

with

1676

a

1677

cou-

1678

pling

1679

CO-

1680

ef-

1681

fi-

1682

cient

1683

which

1684

is

1685

pro-

1686

por-

1687

tional

1688

to

1689

the

1690

mu-

1691

tual

1692

in-

1693

duc-

1694

tance

1695

and

1696

can

1697

be

1698

eval-

1699

u-

1700

ated

1701

in

1702

terms

1703

of

1704

the

1705

qubit

1706

pa-

1707

ram-

1708

e-

1709

ters

1710

[24].

1711

With

1712

the

1713

above

1714

de-

1715

sign

1716

we

1717

then

1718

have

1719

a

1720

struc-

1721

ture

1722

in

1723

close

1724

anal-

1725

ogy

1726

to

1727

the

1728

ion

1729

trap

1730

quan-

1731

tum

1732

com-

1733

puter

1734

in

1735

which

1736

the

1737

qubits

1738

com-

1739

mu-

1740

ni-

1741

cate

1742

through

1743

the

1744

cen-

1745

ter

1746

of

1747

mass

1748

phonon

1749

mode.

1750

To

1751

re-

1752

al-

1753

ize

1754

a

1755

uni-

1756

ver-

1757

sal

1758

quan-

1759

tum

1760

com-

1761

puter,

1762

we

1763

can

1764

use

1765

ei-

1766

ther

1767

the

1768

res-

1769

o-

1770

nant

1771

[13,14]

1772

or

1773

dis-

1774

per-

1775

sive

1776

[13,15,16]

1777

in-

1778

ter-

1779

ac-

1780

tion

1781

be-

1782

tween

1783

the

1784

qubits

1785

and

1786

the

1787

junc-

1788

tion

1789

ar-

1790

ray

1791

mode.

1792

The

1793

in-

1794

ter-

1795

ac-

1796

tion

1797

is

1798

switched

1799

on

1800

and

1801

off

1802

by

1803

tun-

1804

ing

1805

the

1806

en-

1807

ergy

1808

of

1809

the

1810

qubit

1811

into

1812

and

1813

out

1814

of

1815

res-

1816

o-

1817

nance

1818

with

1819

the

1820

junc-

1821

tion

1822

ar-

1823

ray.

1824

Typical

1825

val-

1826

ues

1827

for

1828

ω_p

1829

and

1830

qubit

1831

en-

1832

er-

1833

gies

1834

can

1835

be

1836

cho-

1837

sen

1838

to

1839

be

1840

up

1841

to

1842

10GHz.

1843

The

1844

cou-

1845

pling

1846

strength

1847

g

1848

can

1849

be

1850

tens

1851

of

1852

mega-

1853

hertz

1854

[15,25].

1855

When

1856

the

1857

sys-

1858

tem

1859

scales

1860

up

1861

$(N$

1862

in-

1863

creases),

1864

the

1865

en-

1866

ergy

1867

gap

1868

be-

1869

tween

1870

the

1871

cen-

1872

ter

1873

of

1874

mass

1875

mo-

1876

tion

1877

mode

1878

and

1879

the

1880

higher

1881

modes

1882

should

1883

re-

1884

main

1885

much

1886

greater

1887

than

1888

the

1889

cou-

1890

pling

1891

strength

1892

g

1893

to

1894

avoid

1895

ex-

1896

ci-

1897

ta-

1898

tion

1899

of

1900

up-

1901

per

1902

modes.

1903

When

1904

N

1905

is

1906

large,

1907

the

1908

en-

1909

ergy

1910

dif-

1911

fer-

1912

ence

1913

be-

1914

tween

1915

the

1916

low-

1917

est

1918

two

1919

modes

1920

is

1921

$$\Delta\nu_{01} =$$

1922

$$(\pi^2 K^2/2N^2 \cos \theta^{(0)})\omega_p.$$

1923

For

1924

$K =$

1925

20

1926

[26]

1927

and

1928

$$i_b =$$

1929

0.97,

1930

this

1931

im-

1932

plies

1933

an

1934

up-

1935

per

1936

limit

1937

of

1938

a

1939

few

1940

hun-

1941

dred

1942

for

1943

N.

1944

To

1945

re-

1946

lax

1947

this

1948

limit,

1949

we

1950

can

1951

use

1952

more

1953

com-

1954

pli-

1955

cated

1956

de-

1957

signs

1958

than

1959

the

1960

sim-

1961

ple

1962

1d

1963

ar-

1964

ray

1965

in

1966

Fig.

1967

1

1968

(a).

1969

For

1970

in-

1971

stance,

1972

we

1973

can

1974

con-

1975

sider

1976

a

1977

net-

1978

work

1979

of

1980

N

1981

su-

1982

per-

1983

con-

1984

duct-

1985

ing

1986

is-

1987

lands

1988

in

1989

which

1990

each

1991

pair

1992

of

1993

nodes

1994

is

1995

cou-

1996

pled

1997

by

1998

a

1999

Joseph-

2000

son

2001

junc-

2002

tion,

2003

as

2004

shown

2005

in

2006

Fig.

2007

1

2008

(d).

2009

Each

2010

is-

2011

land

2012

is

2013

still

2014

grounded

2015

though

2016

a

2017

junc-

2018

tion

2019

whose

2020

plasma

2021

fre-

2022

quency

2023

is

2024

$\omega_p,$

2025

and

2026

the

2027

Joseph-

2028

son

2029

en-

2030

ergy

2031

of

2032

the

2033

cou-

2034

pling

2035

junc-

2036

tions

2037

is

2038

K^2

2039

that

2040

of

2041

the

2042

ground-

2043

ing

2044

junc-

2045

tions.

2046

In

2047

this

2048

case,

2049

the

2050

cen-

2051

ter

2052

of

2053

mass

2054

mo-

2055

tion

2056

mode

2057

re-

2058

mains

2059

at

2060

ω_p

2061

and

2062

all

2063

higher

2064

modes

2065

are

2066

pushed

2067

up

2068

to

2069

a

2070

fre-

2071

quency

$$\omega_p \sqrt{1 + NK^2 / \cos \theta^{(0)}}.$$

2073

The

2074

num-

2075

ber

2076

of

2077

junc-

2078

tions

2079

re-

2080

quired

2081

is

2082

on

2083

the

2084

or-

2085

der

2086

$N^2/2$.

2087

In

2088

the

2089

above,

2090

we

2091

take

2092

ad-

2093

van-

2094

tage

2095

of

2096

the

2097

macro-

2098

scopic

2099

quan-

2100

tum

2101

be-

2102

hav-

2103

ior

2104

of

2105

the

2106

Joseph-

2107

son

2108

junc-

2109

tion

2110

phases

2111

to

2112

con-

2113

struct

2114

an

2115

anal-

2116

ogy

2117

of

2118

the

2119

ion

2120

trap

2121

quan-

2122

tum

2123

com-

2124

puter.

2125

We

2126

can

2127

push

2128

this

2129

con-

2130

cept

2131

fur-

2132

ther

2133

and

2134

con-

2135

sider

2136

us-

2137

ing

2138

Joseph-

2139

son

2140

junc-

2141

tion

2142

ar-

2143

rays

2144

to

2145

sim-

2146

u-

2147

late

2148

the

2149

dy-

2150

nam-

2151

ics

2152

of

2153

phys-

2154

i-

2155

cal

2156

sys-

2157

tems.

2158

Con-

2159

sider

2160

a

2161

1d

2162

ar-

2163

ray

2164

of

2165

qubits.

2166

In

2167

the

2168

spin

2169

$1/2$

2170

rep-

2171

re-

2172

sen-

2173

ta-

2174

tion,

2175

the

2176

dy-

2177

nam-

2178

ics

2179

of

2180

the

2181

qubits

2182

are

2183

de-

2184

scribed

2185

by

2186

the

2187

spin

2188

op-

2189

er-

2190

a-

2191

tors

$$S_i^x,$$

2193

S_i^y

2194

and

2195

S_i^z

2196

whose

2197

com-

2198

mu-

2199

ta-

2200

tion

2201

re-

2202

la-

2203

tions

2204

are

2205

de-

2206

fined

2207

by

$$[S_i^\alpha, S_j^\beta] =$$

2209

$$(i/2)\delta_{ij}\epsilon_{\alpha\beta\gamma}S_i^{\gamma}$$

2210

for

2211

$\alpha, \beta, \gamma =$

2212

x, y, z

2213

and

2214

$i, j =$

2215

$0, 1, \dots, N -$

2216

1.

2217

By

2218

us-

2219

ing

2220

a

2221

Jordan-

2222

Wigner

2223

trans-

2224

for-

2225

ma-

2226

tion

2227

[27],

2228

we

2229

can

2230

map

2231

this

2232

qubit

2233

ar-

2234

ray

2235

to

2236

a

2237

col-

2238

lec-

2239

tion

2240

of

2241

spin-

2242

less

2243

fermions

2244

an-

2245

ni-

2246

hi-

2247

lated

2248

by

2249

the

2250

op-

2251

er-

2252

a-

2253

tors

$$f_n = S_n^- K(n), (3)$$

2255

where

2256

$$S_n^\pm =$$

2257

$S_n^x \pm$

2258

iS_n^y

2259

and

2260

$K(n)$

2261

is

2262

a

2263

non-

2264

lo-

2265

cal

2266

op-

2267

er-

2268

a-

2269

tor

2270

de-

2271

fined

2272

by

2273

$$K(n) =$$

$$\exp[i\pi \sum_{m=0}^{n-1} f_m^+ f_m] =$$

2275

$$\exp[i\pi \sum_{m=0}^{n-1}(S_m^z +$$

2276

$1/2)$].

2277

It

2278

can

2279

be

2280

ver-

2281

i-

2282

fied

2283

that

2284

f_n 's

2285

sat-

2286

isfy

2287

the

2288

anti-

2289

commutation

2290

re-

2291

la-

2292

tions

2293

re-

2294

quired

2295

for

2296

fermion

2297

op-

2298

er-

2299

a-

2300

tors.

2301

The

2302

fermion

2303

num-

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ber

2305

op-

2306

er-

2307

a-

2308

tor

2309

is

2310

sim-

2311

ply

2312

$$f_n^\dagger f_n =$$

2313

$S_n^z +$

2314

1/2.

2315

Now,

2316

if

2317

we

2318

cou-

2319

ple

2320

the

2321

qubits

2322

by

2323

near-

2324

est

2325

neigh-

2326

bor

2327

XXZ

2328

in-

2329

ter-

2330

ac-

2331

tions,

2332

the

2333

Hamil-

2334

to-

2335

nian

2336

of

2337

the

2338

sys-

2339

tem

2340

is

2341

$H =$

$$-J_{xy}\sum_i(S_i^xS_{i+1}^x+$$

2343

$$S_i^y S_{i+1}^y)+$$

$$J_z \sum_i S_i^z S_{i+1}^z.$$

2345

Un-

2346

der

2347

the

2348

Jordan-

2349

Wigner

2350

trans-

2351

for-

2352

ma-

2353

tion,

2354

the

2355

sys-

2356

tem

2357

Hamil-

2358

to-

2359

nian

2360

is

2361

trans-

2362

formed

2363

to

$$H = -(J_{xy}/2) \sum_n (f_n^\dagger f_{n+1} + f_{n+1}^\dagger f_n) + J_z \sum_n (f_n^\dagger f_n - 1/2)(f_{n+1}^\dagger f_{n+1} - 1/2). \quad (4)$$

2365

As

2366

can

2367

be

2368

seen

2369

the

2370

cou-

2371

pling

2372

in

2373

the

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XY

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rec-

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tions

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trans-

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forms

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into

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hop-

2382

ing

2383

of

2384

the

2385

fermions

2386

be-

2387

tween

2388

neigh-

2389

bor-

2390

ing

2391

sites

2392

and

2393

the

2394

Ising

2395

cou-

2396

pling

2397

causes

2398

near-

2399

est

2400

neigh-

2401

bor

2402

in-

2403

ter-

2404

ac-

2405

tions

2406

be-

2407

tween

2408

the

2409

fermions.

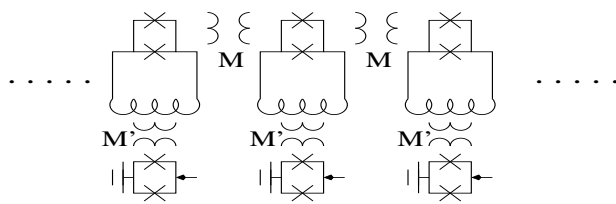


Fig. 2.

A
1d
rf-SQUID
qubit
ar-
ray
in-
duc-
tively
cou-
pled
to
lo-
cal
Joseph-
son
junc-
tion
os-
cil-
la-
tors.
The
dc-SQUIDs
of
the
qubits
are
in-
duc-
tively
cou-
pled

2411

in-

2412

ter-

2413

est-

2414

ing

2415

Sce-

2416

nario

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arises

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when

2419

we

2420

cou-

2421

ple

2422

a

2423

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qubit

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ray

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and

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“phonon

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cil-

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Joseph-

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Fig.

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electron-

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electron

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tions.

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the

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and

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2509

fine

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con-

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and

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fermion

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2589

sys-

2590

tem

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elec-

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tron

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is

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served;

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ever

2601

a

2602

$$B^x \sigma^x$$

2603

term

2604

of

2605

the

2606

qubit

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can

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flip

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2614

fined

2615

by

2616

the

2617

eigen-

2618

states

2619

of

2620

σ^z .

2621

Ac-

2622

cord-

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Jordan-

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qubit

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fermion.

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2689

pro-

2690

pri-

2691

ately

2692

so

2693

that

2694

the

2695

B^x

2696

field

2697

of

2698

the

2699

qubit

2700

is

2701

much

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smaller

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[15]

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than

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other

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ergy

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scales

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of

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the

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sys-

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and

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for

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time

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scale

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of

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in-

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To

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fur-

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ther

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press

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fermion

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cre-

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ation

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and

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an-

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ni-

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hi-

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tion,

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we

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may

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choose

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B^z

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to

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be

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large,

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makes

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changes

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for-

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bid-

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den.

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An-

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point

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is

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pling

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qubit

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and

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the

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Joseph-

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son

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phase

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should

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to

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change

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ta-

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tions,

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since

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in

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a

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phys-

2800

i-

2801

cal

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sys-

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tem

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no

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pro-

2806

cess

2807

hap-

2808

pens

2809

in

2810

which

2811

an

2812

elec-

2813

tron

2814

is

2815

cre-

2816

ated

2817

(an-

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ni-

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hi-

2820

lated)

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and

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a

2823

phonon

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is

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an-

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hi-

2828

lated

2829

(cre-

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ated).

2831

In

2832

the

2833

fol-

2834

low-

2835

ing

2836

we

2837

fo-

2838

cus

2839

on

2840

the

2841

1d

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Hol-

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stein

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model

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of

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spin-

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less

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elec-

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trons,

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of

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both

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i-

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cal

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and

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prac-

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ti-

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est

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[30,31].

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The

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Hamil-

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to-

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nian

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of

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sys-

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tem

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is

$$H = -t \sum_i (c_i^\dagger c_{i+1} + c_{i+1}^\dagger c_i) + \omega \sum_i a_i^\dagger a_i - g \sum_i (c_i^\dagger c_i - 1/2)(a_i + a_i^\dagger), \quad (5)$$

2878

where

2879

c_i

2880

de-

2881

stroys

2882

a

2883

fermion

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at

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site

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i

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and

2888

a_i

2889

de-

2890

stroys

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a

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lo-

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cal

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phonon

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of

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fre-

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quency

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ω .

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This

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sys-

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tem

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fill-

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the

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tio

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g/ω .

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When

2918

$g <$

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$g_c,$

2920

the

2921

ground

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state

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has

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metal-

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lic

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(Lut-

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uid)

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phase.

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When

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g

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ex-

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ceeds

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$g_c,$

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it

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ex-

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hibits

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an

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in-

2942

su-

2943

lat-

2944

ing

2945

phase

2946

with

2947

charge

2948

den-

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sity

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wave

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(CDW)

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long

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range

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or-

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der

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[30,31].

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Though

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it

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can

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be

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solved

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ters

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cu-

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of

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the

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crit-

2990

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2991

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cou-

2993

pling

2994

strength

2995

and

2996

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ergy

2998

gaps

2999

is

3000

dif-

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fi-

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cult

3003

and

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only

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lim-

3006

ited

3007

size

3008

sys-

3009

tems

3010

at

3011

half

3012

fill-

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3014

have

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been

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model

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3043

ing

3044

a

3045

su-

3046

per-

3047

con-

3048

duct-

3049

ing

3050

qubit

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ar-

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ray

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and

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lo-

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cal

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OS-

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cil-

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tor

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modes

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Joseph-

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SQUID

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whose

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SQUIDS

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The

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3105

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3106

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3107

re-

3108

al-

3109

ized

3110

by

3111

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3112

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3113

a

3114

flux

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former

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3119

rupted

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dc-

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SQUID

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whose

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rent

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can

3131

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tuned

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by

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a

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flux

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bias,

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as

3138

dis-

3139

cussed

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in

3141

[24,28,29].

3142

As

3143

a

3144

re-

3145

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of

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this

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inter-

3149

qubit

3150

cou-

3151

pling

3152

and

3153

the

3154

qubit

3155

bias,

3156

the

3157

qubit

3158

ar-

3159

ray

3160

Hamil-

3161

to-

3162

nian

3163

is

3164

$$H_Q =$$

$$-\sum_i B^z S_i^z -$$

$$J \sum_i S_i^x S_{i+1}^x,$$

3167

where

3168

B^z

3169

is

3170

de-

3171

ter-

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mined

3173

by

3174

the

3175

flux

3176

bias

3177

of

3178

the

3179

qubits

3180

and

3181

J

3182

is

3183

pro-

3184

por-

3185

tional

3186

to

3187

the

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tance

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M.

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The

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phonon

3200

modes

3201

are

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re-

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al-

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ized

3205

by

3206

Joseph-

3207

son

3208

junc-

3209

tions

3210

whose

3211

OS-

3212

cil-

3213

la-

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$$Q^2/2C_J+$$

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$\Phi^2/2L' -$

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$$(I_c/24)(2\pi/\Phi_0)^3\Phi^4,$$

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Friedman,

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Tolpygo,

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Harmans,

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Anderson,

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Lobb,

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Wagner,

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Meyer,

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Smirnov,

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(2004).

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Bertet,

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Nakamura,

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(London)

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(2004).

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Feldman,

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Blais,

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Schoelkopf,

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Phys.

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(2004).

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[17]

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Wallraff,

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Schuster,

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Blais,

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Frunzio,

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S.

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Huang,

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J.

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Majer,

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Kumar,

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S.

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M.

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Girvin,

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and

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Schoelkopf,

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Nature

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(London)

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431,

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162

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(2004).

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Xu,

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W.

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Strauch,

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Dutta,

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Johnson,

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R.

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Ramos,

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A.

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J.

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Berkley,

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H.

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Paik,

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J.

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R.

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Anderson,

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A.

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J.

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Dragt,

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C.

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J.

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Lobb,

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and

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Wellstood,

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Phys.

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Rev.

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Lett.

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(2005).

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Kadin,

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“Introduction

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to

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superconducting

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circuits,”

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Wiley-

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Interscience,

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1999.

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B.

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A.

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Mazin,

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P.

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K.

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Day,

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Н.

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G.

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LeDuc,

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A.

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Vayonakis,

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and

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J.

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Zmuidzinas,

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Proc.

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SPIE

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283

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Day,

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G.

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Leduc,

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Mazin,

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Vayonakis,

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and

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J.

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Zmuidzinas,

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Nature

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425,

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N.

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Rando,

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P.

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Videler,

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A.

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Peacock,

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A.

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van

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Dordrecht,

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P.

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Verhoeve,

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R.

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Venn,

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A.

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C.

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Wright,

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and

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J.

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Lumley,

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J.

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Appl.

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Phys.

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(1995).

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J.

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Cirac

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Zoller,

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Rev.

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Lett.

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(1995).

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Orlando,

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Mooij,

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Levitov,

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S.

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Lloyd,

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J.

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Mazo,

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Phys.

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Rev.

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Lam,

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Z.

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Zheng,

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Phys.

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Rev.

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B

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(2001).

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J.

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You,

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J.

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S.

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Tsai,

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Nori,

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Phys.

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Rev.

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Lett.

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197902

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(2002).

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Jordan

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Wigner,

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Phys.

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J.

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Mooij,

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T.

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Orlando,

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Levitov,

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C.

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Н.

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Wal,

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and

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Science

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285,

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V.

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Filippov,

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S.

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Tolpygo,

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J.

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Mannik,

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and

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J.

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Lukens,

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Trans.

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Appl.

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Supercond.

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R.

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McKenzie,

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C.

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J.

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Hamer,

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D.

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Murray,

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Phys.

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Rev.

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9676

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(1996).

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Bursill,

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McKenzie,

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Hamer,

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Phys.

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Lett.

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(1998).

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Freericks,

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M.

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Jarrell,

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G.

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Mahan,

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Phys.

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Rev.

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Lett.

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